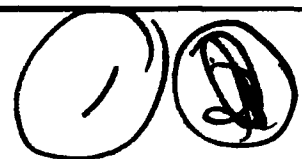


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# VALIDITY MEASURES IN THE CONTEXT OF LATENT TRAIT MODELS

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## I Introduction

In classical mental test theory, the reliability and the validity coefficients of a test are considered to be two essential topics. In modern mental test theory, or in latent trait models, this is not the case, however. In particular, test validity is one concept that has been neglected in the context of latent trait models.

Several types of validity have been identified and discussed in classical mental test theory, which include content validity, construct validity, and criterion-oriented validity. Perhaps we can say that, in modern mental test theory, both content validity and construct validity are well accommodated, although they are not explicitly stated. If each item is based upon cognitive processes that are directly related to the ability to be measured, then the content of the operationally defined latent variable behind the examinees' performances will be validated. Also construct validity can be identified, with all the mathematically sophisticated structures and functions which characterize latent trait models and which classical mental test theory does not provide.

With respect to the criterion-oriented validity, however, so far latent trait models have not offered so much as they did to the test reliability and to the standard error of measurement (cf. Samejima, 1977, 1990). From the scientific point of view, however, we need to confirm if, indeed, the test measures what it is supposed to measure, even if we have chosen our items carefully enough in regard to their contents, and even if we are equipped with highly sophisticated mathematics.

In classical mental test theory, the validity coefficient is a single number, i.e., the product-moment correlation coefficient between the test score and the criterion variable. Researchers tend to put too much faith in the validity coefficient, or in the reliability coefficient, however. The correlation coefficient is largely affected by the heterogeneity of the group of examinees, i.e., for a fixed test the coefficient tends to be higher when individual differences among the examinees in the group are greater, and vice versa (cf. Samejima, 1977). Thus we must keep in mind that *so-called* test validity represents the degree of heterogeneity in ability among the examinees tested, as well as the quality of the test itself.

By virtue of the population-free nature of latent trait theory, we should be able to find some indices of item validity, and of test validity, which are not affected by the group of examinees. The resulting indices should not be *incidental* as those in classical mental test theory are, but truly be attributes of the item and the test themselves.

In the present research an attempt has been made to obtain such population-free measures of item validity and of test validity, which are basically *locally* defined.

## II Performance Function: Regression of the External Criterion Variable on the Latent Variable

Let  $\theta$  be ability, or latent trait, which assumes any real number. We assume that there is a set of  $n$  test items measuring  $\theta$  whose characteristics are known. Let  $g$  denote such an item,  $k_g$  be a discrete item response to item  $g$ , and  $P_{k_g}(\theta)$  denote the *operating characteristic* of  $k_g$ , or the conditional probability assigned to  $k_g$ , given  $\theta$ , i.e.,

$$(2.1) \quad P_{k_g}(\theta) = \text{Prob.}[k_g | \theta] .$$

We assume that  $P_{k_g}(\theta)$  is three-times differentiable with respect to  $\theta$ . We have for the *item response information function*

$$(2.2) \quad I_{k_g}(\theta) = -\frac{\partial^2}{\partial \theta^2} \log P_{k_g}(\theta) ,$$

and the *item information function* is defined as the conditional expectation of  $I_{k_g}(\theta)$  , given  $\theta$  , so that we can write

$$(2.3) \quad I_g(\theta) = E[I_{k_g}(\theta) | \theta] = \sum_{k_g} I_{k_g}(\theta) P_{k_g}(\theta) .$$

In the special case where the item  $g$  is scored dichotomously, this item information function is simplified to become

$$(2.4) \quad I_g(\theta) = \left[ \frac{\partial}{\partial \theta} P_g(\theta) \right]^2 [P_g(\theta) \{1 - P_g(\theta)\}]^{-1} ,$$

where  $P_g(\theta)$  is the operating characteristic of the correct answer to item  $g$  .

Let  $V$  be a response pattern such that

$$(2.5) \quad V = \{ k_g \}' \quad g = 1, 2, \dots, n .$$

The operating characteristic,  $P_V(\theta)$  , of the response pattern  $V$  is defined as the conditional probability of  $V$  , given  $\theta$  , and by virtue of *local independence* we can write

$$(2.6) \quad P_V(\theta) = \prod_{k_g \in V} P_{k_g}(\theta) .$$

The *response pattern information function*,  $I_V(\theta)$  , is given by

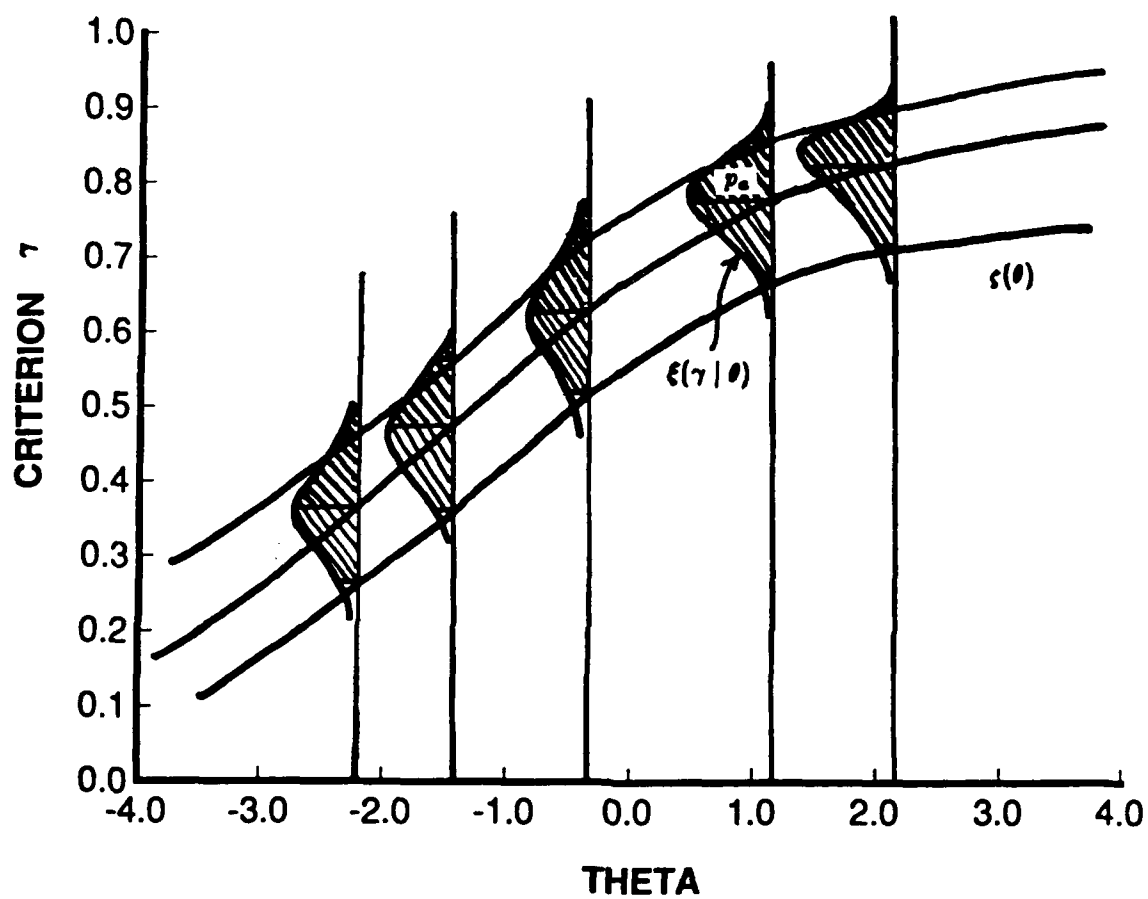
$$(2.7) \quad I_V(\theta) = -\frac{\partial^2}{\partial \theta^2} \log P_V(\theta) = \sum_{k_g \in V} I_{k_g}(\theta) ,$$

and the *test information function*,  $I(\theta)$  , is defined as the conditional expectation of  $I_V(\theta)$  , given  $\theta$  , and we obtain from (2.2), (2.3), (2.5), (2.6) and (2.7)

$$(2.8) \quad I(\theta) = E[I_V(\theta) | \theta] = \sum_V I_V(\theta) P_V(\theta) = \sum_{g=1}^n I_g(\theta) .$$

A big advantage of the modern mental test theory is that the standard error of estimation can *locally* be defined by using  $[I(\theta)]^{-1/2}$  . Unlike in classical mental test theory this function *does not depend upon the population of examinees*, but is solely a property of the test itself, which should be the way if we call it the standard error, or the reliability, of a test. It is well known that this function provides us with the asymptotic standard deviation of the conditional distribution of the maximum likelihood estimate of  $\theta$  , given its true value.

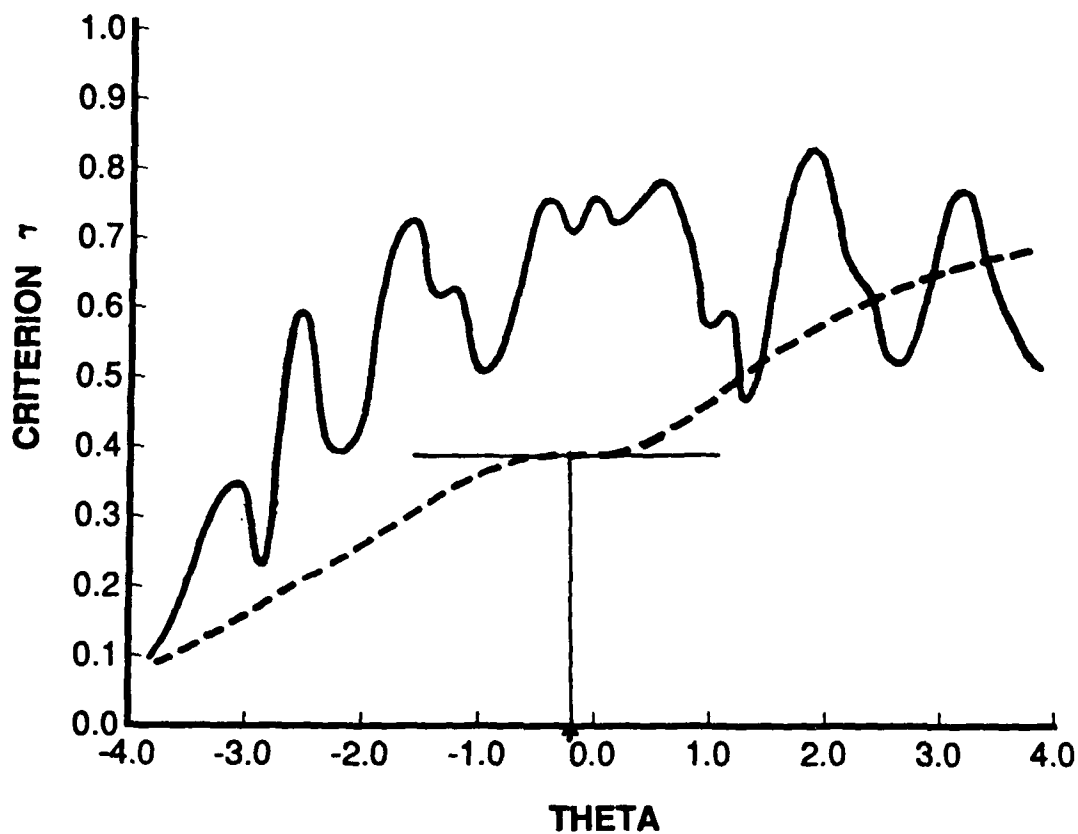
It is assumed that there exists an external criterion variable, which can be measured directly or indirectly. This is the situation which is also assumed when we deal with *criterion-oriented validity* or *predictive validity* in classical mental test theory.



0.780 0.80 1.80 8.80 8.00  
DUMMY.DAT, INDIRAM, plotted by LEE HAI QAM

FIGURE 2-1

Relationships among  $\theta$ ,  $\gamma$ ,  $p_\gamma$ ,  $\xi(\gamma | \theta)$  and  $s(\theta)$ .



0.750 0.50 1.00 0.50 0.00  
DUMMY.DAT, IMPRESS1, plotted by F. SAME/MA

FIGURE 2-2

Two Hypothetical Performance Functions  $\zeta(\theta)$ , One of Which Is Not Likely to Be the Case (Solid Line), and the Other Has a Derivative Equal to Zero at One Point of  $\theta$  (Dashed Line).



Let  $\gamma$  denote the *criterion variable*, representing the performance in a specific job, etc. We shall consider the conditional density of the criterion performance, given ability, and denote it by  $\xi(\gamma | \theta)$ . The *performance function*,  $\zeta(\theta)$ , can be defined as the regression of  $\gamma$  on  $\theta$ , or by taking, say, the 75, 90 or 95 percentile point of each conditional distribution of  $\gamma$ , given  $\theta$ . Let  $p_a$  denote the probability which is large enough to satisfy us as a confidence level. Thus we can write

$$(2.9) \quad p_a = \int_{\zeta(\theta)}^{\bar{\gamma}} \xi(\gamma | \theta) d\gamma ,$$

where  $\bar{\gamma}$  denotes the least upper bound of the criterion variable  $\gamma$ .

Figure 2-1 illustrates the relationships among  $\theta$ ,  $\gamma$ ,  $p_a$ ,  $\xi(\gamma | \theta)$  and  $\zeta(\theta)$ . It may be reasonable to assume that the functional relationship between  $\theta$  and  $\zeta(\theta)$  is relatively simple, not as is illustrated by the solid line in Figure 2-2, i.e., we do not expect  $\zeta(\theta)$  to go up and down frequently within a relatively short range of  $\theta$ . We shall assume that  $\zeta(\theta)$  is twice differentiable with respect to  $\theta$ .

In dealing with an additional dimension or dimensions, i.e., the criterion variable or variables, in latent space, one of the most difficult things is to keep the population-free nature, which is characteristic of the latent trait models, the main feature that distinguishes the theory from classical mental test theory, among others. If we consider the projection of the operating characteristic of a discrete item response on the criterion dimension, for example, then the resulting operating characteristic as a function of  $\gamma$  has to be *incidental*, for it has to be affected by the population distribution of  $\theta$ .

We need to start from the conditional distribution of  $\gamma$ , given  $\theta$ , therefore, which can be conceived of as being intrinsic in the relationship between the two variables, and independent of the population distribution of  $\theta$ .

We assume that  $\zeta(\theta)$  takes on the same value only at a finite or an enumerable number of points of  $\theta$ . Let  $P_{k_g}^*(\zeta)$  be the conditional probability assigned to the discrete response  $k_g$ , given  $\zeta$ . We can write

$$(2.10) \quad P_{k_g}^*(\zeta) = \sum_{\zeta(\theta)=\zeta} P_{k_g}(\theta) .$$

### III When $\zeta(\theta)$ Is Strictly Increasing in $\theta$ : Simplest Case

#### [III.1] Amounts of Item and Test Information for a Fixed Value of $\zeta$

The simplest case is that  $\zeta(\theta)$  is strictly increasing in  $\theta$ . In this case,  $\zeta(\theta)$  has a one-to-one correspondence with  $\theta$ , and (2.10) becomes simplified into the form

$$(3.1) \quad P_{k_g}^*(\zeta) = P_{k_g}^*[\zeta(\theta)] = P_{k_g}(\theta) .$$

If, in addition,  $\partial\theta/\partial\zeta$  is finite throughout the entire range of  $\theta$ , then we obtain

$$(3.2) \quad \frac{\partial}{\partial\zeta} P_{k_g}^*(\zeta) = \left\{ \frac{\partial}{\partial\theta} P_{k_g}(\theta) \right\} \frac{\partial\theta}{\partial\zeta} .$$

Let  $I_{k_g}^*(\zeta)$  be the item response information function defined as a function of  $\zeta$ . We can write

$$(3.3) \quad I_{k_g}^*(\zeta) = -\frac{\partial^2}{\partial\zeta^2} \log P_{k_g}^*(\zeta) = -\frac{\partial}{\partial\zeta} \left[ \left\{ \frac{\partial}{\partial\theta} \log P_{k_g}(\theta) \right\} \frac{\partial\theta}{\partial\zeta} \right]$$

$$= I_{k_g}(\theta) \left( \frac{\partial \theta}{\partial \zeta} \right)^2 - \left[ \frac{\partial}{\partial \theta} P_{k_g}(\theta) \right] [P_{k_g}(\theta)]^{-1} \frac{\partial^2 \theta}{\partial \zeta^2} .$$

Let  $I_g^*(\zeta)$  and  $I^*(\zeta)$  be the amounts of information given by a single item  $g$  and by the total test, respectively, for a fixed value of  $\zeta$ . Then we have from (2.3), (2.8) and (3.3)

$$(3.4) \quad I_g^*(\zeta) = E[I_{k_g}^*(\zeta) | \zeta] = \sum_{k_g} I_{k_g}^*(\zeta) P_{k_g}^*(\zeta) = I_g(\theta) \left( \frac{\partial \theta}{\partial \zeta} \right)^2$$

and

$$(3.5) \quad I^*(\zeta) = \sum_{g=1}^n I_g^*(\zeta) = I(\theta) \left( \frac{\partial \theta}{\partial \zeta} \right)^2 .$$

If we take the square roots of these two information functions defined for  $\zeta$ , then we obtain

$$(3.6) \quad [I_g^*(\zeta)]^{1/2} = [I_g(\theta)]^{1/2} \frac{\partial \theta}{\partial \zeta}$$

and

$$(3.7) \quad [I^*(\zeta)]^{1/2} = [I(\theta)]^{1/2} \frac{\partial \theta}{\partial \zeta} .$$

Since a certain constant nature exists for the square root of the item information function while the same is not true with the original item information function (cf. Samejima, 1979, 1982),  $[I_g^*(\zeta)]^{1/2}$  given by (3.6) instead of the original function given by (3.4) may be more useful in some occasions. This will be discussed later in this section, when the validity in selection *plus* classification is discussed.

### [III.2] Validity in Selection

Suppose that we have a critical value,  $\gamma_0$ , of the criterion variable, which is needed for succeeding in a specified job, and that we try to accept applicants whose values of the criterion variable are  $\gamma_0$  or greater. If our primary purpose of testing is to make an accurate selection of applicants, then (3.6) and (3.7) for  $\zeta = \gamma_0$ , or their squared values shown by (3.4) and (3.5), indicate item and test validities, respectively. In other words, if for some item formula (3.6) or (3.4) assumes high values at  $\zeta = \gamma_0$ , then the standard error of estimation of  $\zeta$  around  $\zeta = \gamma_0$  becomes small and chances are slim that we make misclassifications of the applicants by accepting unqualified persons and rejecting qualified ones, and vice versa. The same logic applies to the total test by using formula (3.7) or (3.5) instead of (3.6) or (3.4).

It should be noted in (3.6) or in (3.7), that  $[I_g^*(\tau)]^{1/2}$  or  $[I^*(\tau)]^{1/2}$  consists of two factors, i.e., 1) the square root of the item information function  $I_g(\theta)$  or that of the test information function  $I(\theta)$  and 2) the partial derivative of ability  $\theta$  with respect to  $\zeta$  at  $\zeta = \gamma_0$ . These two factors in each formula are independent of each other, i.e., one belongs to the item or to the test and the other to the statistical relationship between  $\theta$  and  $\gamma$ . We also notice that these two factors are in a supplementary relationship, i.e., even if one assumes a small value the other can supplement it in order to make the resulting product large. Thus while it is important to have a large amount of item information, or of test information, it is even more so to have large values of the derivative,  $\partial \theta / \partial \zeta$ , in the vicinity of  $\zeta = \gamma_0$ , for this will increase the amount of item information defined with respect to  $\zeta$  uniformly in that vicinity, and also that of test information, as is obvious from the right hand sides of (3.6) and

(3.7). In other words, it is desirable for the purpose of selection for  $\zeta$  to increase slowly in  $\theta$  in the vicinity of  $\zeta = \gamma_0$ .

Since, in general, the same ability  $\theta$  has predictabilities for more than one kind of job performance, or of potential of achievement, *the performance function varies for different criterion variables*. Note that neither  $[I_\theta(\theta)]^{1/2}$  nor  $[I(\theta)]^{1/2}$  is changed even when the criterion variable is switched. Thus, for a fixed item or test whose amount of information is reasonably large around  $\zeta = \gamma_0$ , the derivative  $\partial\theta/\partial\zeta$  in the vicinity of  $\zeta = \gamma_0$  determines the appropriateness of the use of the item or of the test for the purpose of selection with respect to a specific job, etc. If this derivative assumes a high value, then an item or a test which provides us with a medium amount of information may be acceptable for our purpose of selection, while we will need an item or a test whose amount of information is substantially larger if the derivative is low. Also for the same criterion variable  $\gamma$  the derivative  $\partial\theta/\partial\zeta$  varies for different values of  $\gamma_0$ , so the appropriateness of an item or of a test depends upon our choice of  $\gamma_0$ , too.

The above logic also applies for the formulae (3.4) and (3.5), i.e., for the case in which we choose the information functions, instead of their square roots, changing  $\partial\theta/\partial\zeta$  to its squared value.

It is obvious from (3.4) and (3.6) that we can choose either  $I_\theta(\theta(\gamma_0))$  or  $[I_\theta(\theta(\gamma_0))]^{1/2}$  for use in item selection, for their rank orders across different items are identical, and they equal the rank orders of  $I_\theta^*(\gamma_0)$  as well as those of  $[I_\theta^*(\gamma_0)]^{1/2}$ .

### [III.3] Validity in Selection Plus Classification

If we take another standpoint that our purpose of testing is not only to make a right selection of applicants but also to predict the degree of success in the job for each selected individual, then we will need to integrate  $[I_\theta^*(\zeta)]^{1/2}$  and  $[I^*(\zeta)]^{1/2}$ , respectively, since we must estimate  $\zeta$  accurately not only around  $\zeta = \gamma_0$  but also for  $\zeta > \gamma_0$ . If we choose  $[I_\theta^*(\zeta)]^{1/2}$  and  $[I^*(\zeta)]^{1/2}$  in preference to their squared values, we will obtain from (3.6) and (3.7)

$$(3.8) \quad \int_{\Omega_\zeta} [I_\theta^*(\zeta)]^{1/2} d\zeta = \int_{\Omega_\theta} [I_\theta(\theta)]^{1/2} d\theta$$

and

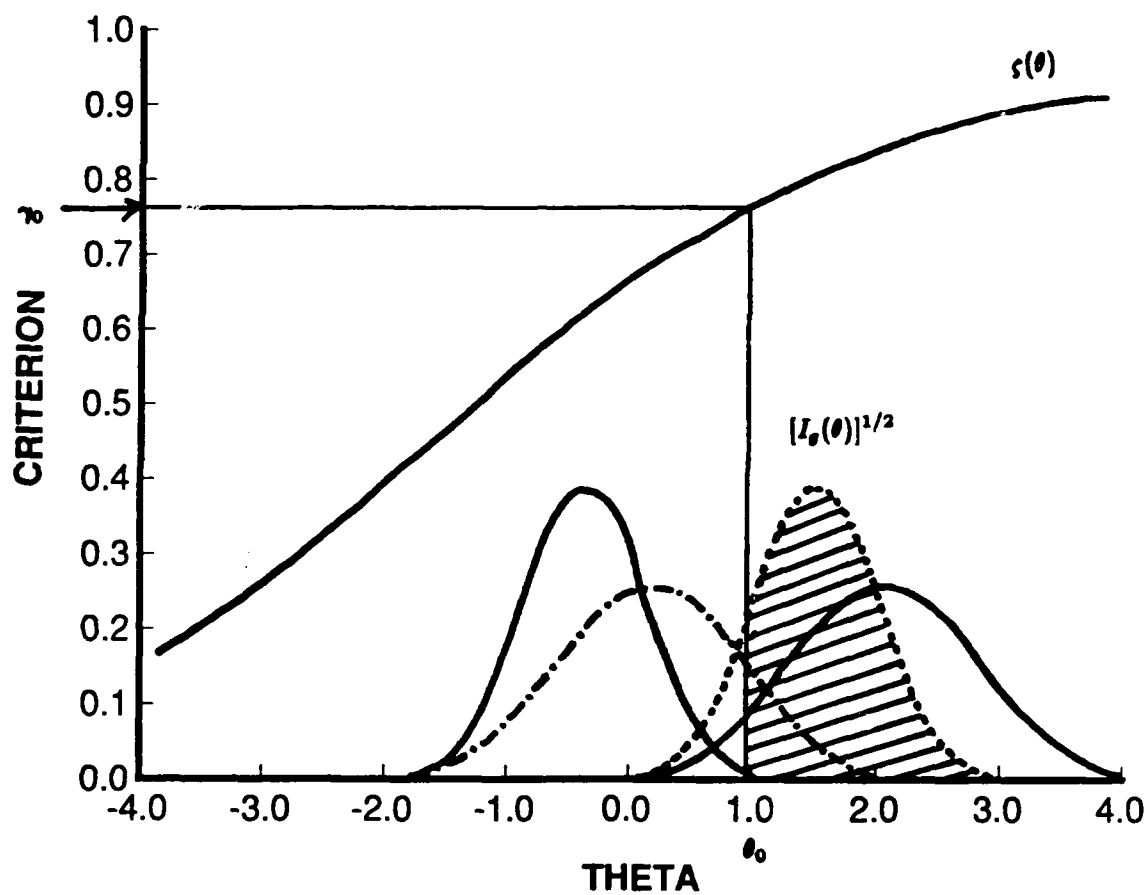
$$(3.9) \quad \int_{\Omega_\zeta} [I^*(\zeta)]^{1/2} d\zeta = \int_{\Omega_\theta} [I(\theta)]^{1/2} d\theta,$$

where  $\Omega_\zeta$  and  $\Omega_\theta$  indicate the domains of  $\zeta$  and  $\theta$  for which  $\zeta(\theta) \geq \gamma_0$ , respectively. In other words, when our purpose of testing is not only to make an accurate selection among the applicants but also to discriminate their ability accurately for future purposes among those who were accepted with respect to the criterion variable  $\gamma$ , we need to select items which assume high values of (3.8) instead of (3.6), or a test which provides us with a high value of (3.9) in place of (3.7).

Note that formulae (3.8) and (3.9) imply that we can obtain these two validity measures directly from the original item and test information functions, respectively, i.e., *without actually transforming*  $\theta$  to  $\zeta$ , as long as we can identify the domain  $\Omega_\theta$ . This is true for any criterion variable  $\gamma$ .

Some examples illustrating the values of (3.8) are given in Figure 3-1 for hypothetical items. In the simplest case observed in this section and illustrated in Figures 2-1 and 3-1, these two domains,  $\Omega_\theta$  and  $\Omega_\zeta$ , are provided by the two intervals,  $(\theta_0, \infty)$  and  $(\gamma_0, \bar{\gamma})$ , where

$$(3.10) \quad \theta_0 = \theta(\gamma_0)$$



0.750 0.50 1.50 6.50 8.00  
DUMMY.DAT, INDUMM, plotted by LEE HAI GAN

FIGURE 3-1

Some Examples of the Relationship between  $\gamma_0$  and the Item Validity Measure  
Given by (3.8).

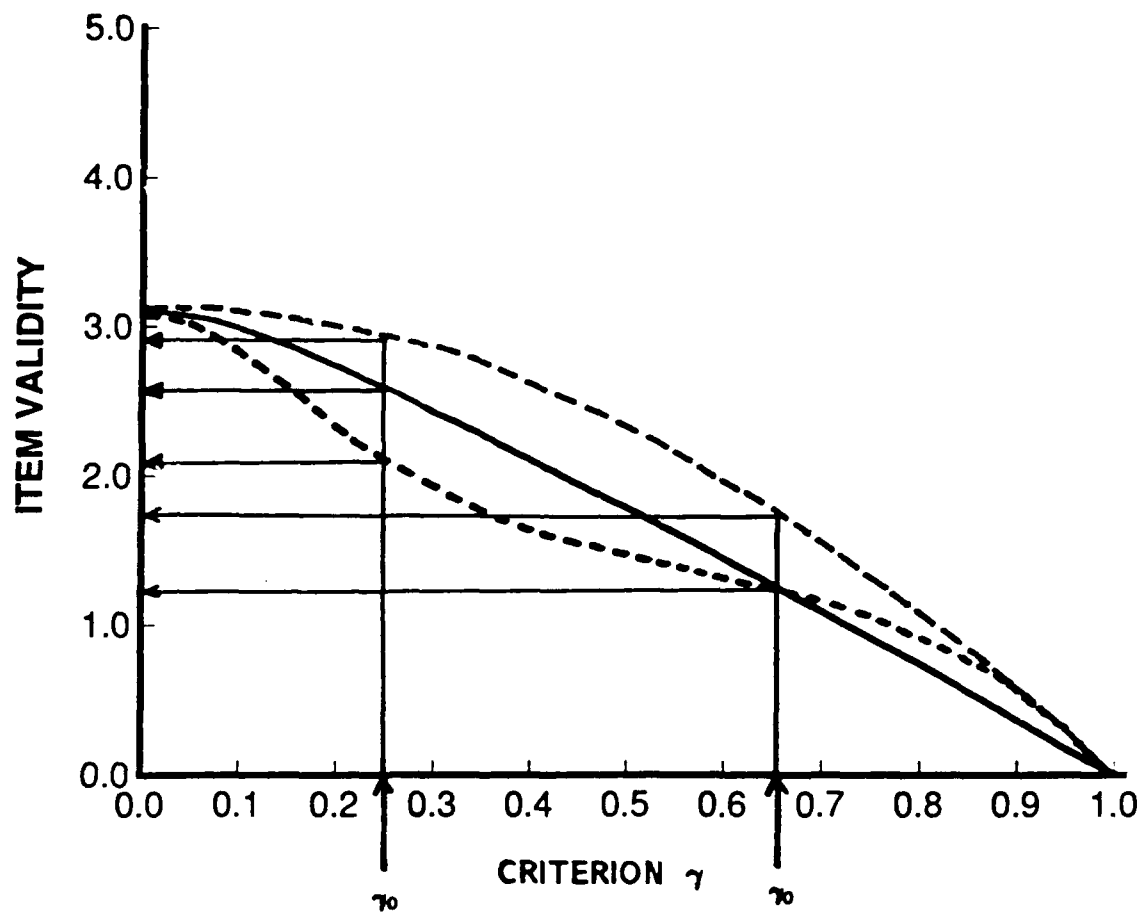


FIGURE 3-2

Relationship between  $\gamma_0$  and Item Validity Indicated by (3.8) for Three Hypothetical Dichotomous Items Whose Operating Characteristics for the Correct Answer Are Strictly Increasing with Zero and Unity as Their Asymptotes.

and  $\bar{\gamma}$  denotes the least upper bound of  $\gamma$ .

It should be noted that this pair of validity measures depends upon our choice of the critical value  $\gamma_0$ . If this value is low, i.e., a specified job does not require high levels of competence with respect to the criterion variable  $\gamma$ , then these validity indices assume high values, and vice versa. It has been pointed out (Samejima, 1979, 1982) that there is a certain constancy in the amount of information provided by a single test item. To give some examples, if an item is dichotomously scored and has a strictly increasing operating characteristic for success with zero and unity as its two asymptotes, then the area under the curve for  $[I_\theta(\theta)]^{1/2}$  equals  $\pi$ , regardless of the mathematical form of the operating characteristic and its parameter values; if it follows a three-parameter model with the lower asymptote,  $c_\theta (> 0)$ , then this area is less than  $\pi$  and strictly decreasing in, and solely dependent upon,  $c_\theta$ . We can see, therefore, that if our items belong to the first type then the functional relationship between  $\gamma_0$  and the item validity measure given by (3.8) will be *monotone decreasing*, with  $\pi$  and zero as its two asymptotes, for each and every item. Figure 3-2 illustrates this relationship for three hypothetical items of this type. As we can see in this figure, the appropriateness of the items changes with  $\gamma_0$  in an absolute sense, and also relatively to other items with  $\gamma_0$ , and the rank orders of desirability among the items depend upon our choice of  $\gamma_0$ .

We can see from (3.8) that this validity measure necessarily assumes a high value if an item is difficult, and the same applies to (3.9) for the total test. This implies that these validity measures alone cannot indicate the desirability of an item and of a test precisely for a specific population of examinees. In selecting items or a test, therefore, it is desirable to take the ability distribution of the examinees into account, if the information concerning the ability distribution of a target population is more or less available. In so doing we shall be able to avoid choosing items which are too difficult for the target population of examinees.

Let  $f(\theta)$  denote the density function of the ability distribution for a specific population of examinees, and  $f^*(\zeta)$  be that of  $\zeta$  for the same population. Then we can write

$$(3.11) \quad f^*(\zeta) = f(\theta) \frac{\partial \theta}{\partial \zeta}.$$

Adopting this as the weight function, from (3.6) and (3.7) we obtain as the validity indices tailored for a specific population of examinees

$$(3.12) \quad \int_{\Omega_\zeta} [I_\theta^*(\zeta)]^{1/2} f^*(\zeta) d\zeta = \int_{\Omega_\theta} [I_\theta(\theta)]^{1/2} f(\theta) \frac{\partial \theta}{\partial \zeta} d\theta$$

and

$$(3.13) \quad \int_{\Omega_\zeta} [I^*(\zeta)]^{1/2} f^*(\zeta) d\zeta = \int_{\Omega_\theta} [I(\theta)]^{1/2} f(\theta) \frac{\partial \theta}{\partial \zeta} d\theta.$$

Thus by using (3.12) and (3.13) instead of (3.8) and (3.9) we shall be able to make appropriate item selection and test selection for a target population or sample, provided that the information concerning its ability distribution is more or less available. Note that, unlike (3.8) and (3.9), we need  $\partial \theta / \partial \zeta$  in evaluating these measures given by (3.12) and (3.13). Thus not only are these validity measures specific for the ability distribution of a target population, but also they are heavily dependent upon the functional formula of  $\zeta(\theta)$ .

If we choose to use the area under the curve of the information function instead of that of its square root, we obtain from (3.4) and (3.5)

$$(3.14) \quad \int_{\Omega_\zeta} I_\theta^*(\zeta) d\zeta = \int_{\Omega_\theta} I_\theta(\theta) \frac{\partial \theta}{\partial \zeta} d\theta$$

and

$$(3.15) \quad \int_{\Omega_\zeta} I^*(\zeta) d\zeta = \int_{\Omega_\theta} I(\theta) \frac{\partial \theta}{\partial \zeta} d\theta ,$$

respectively. We notice that in this case, unlike those of (3.8) and (3.9), the integrands of the right hand sides of (3.14) and (3.15) are no longer independent of the functional formula of  $\zeta(\theta)$ . Also when information about the ability distribution of a target population of examinees is more or less available, the "tailored" item and test validity indices become

$$(3.16) \quad \int_{\Omega_\zeta} I_\theta^*(\zeta) f^*(\zeta) d\zeta = \int_{\Omega_\theta} I_\theta(\theta) f(\theta) \left(\frac{\partial \theta}{\partial \zeta}\right)^2 d\theta$$

and

$$(3.17) \quad \int_{\Omega_\zeta} I^*(\zeta) f^*(\zeta) d\zeta = \int_{\Omega_\theta} I(\theta) f(\theta) \left(\frac{\partial \theta}{\partial \zeta}\right)^2 d\theta ,$$

respectively, if we choose to use the information functions instead of their square roots.

Note that, unlike the validity measures for "selection" purposes, in the present situation the rank orders of validity across different items, or different tests, depend upon the choice of the validity index. Thus a question is: which of the formulae, (3.8) or (3.14), and (3.9) or (3.15), are better as the item and the test validity indices for "selection plus classification" purposes? A similar question is also addressed with respect to (3.12) and (3.16), and to (3.13) and (3.17). These are tough questions to answer. While the choice of the square root of the item information function has an advantage of a certain constancy which has been observed earlier in this subsection, the use of the item information has a benefit of additivity, i.e., by virtue of (2.8) the sum total of (3.14) over all the item  $g$ 's equals (3.15), and the same relationship holds between (3.16) and (3.17). The answers to these questions are yet to be searched.

### [III.4] Validity in Classification

When our purpose of testing is strictly the classification of individuals, as in assigning those people to different training programs, in guidance, etc., (3.8) and (3.9), or (3.14) and (3.15), also serve as the validity measures of an item and of a test, respectively. In this case, we must set  $\gamma_0 = \underline{\gamma}$  in defining the domains,  $\Omega_\zeta$  and  $\Omega_\theta$ , where  $\underline{\gamma}$  is the greatest lower bound of  $\gamma$ . Thus the two domains,  $\Omega_\zeta$  and  $\Omega_\theta$ , in these formulae become those of  $\zeta$  and  $\theta$  for which  $\underline{\gamma} \leq \zeta(\theta) \leq \bar{\gamma}$ . It is obvious that these formulae provide us with the item and the test validity measures, respectively, for the same reason explained in [III.3].

The same logic applies for the "tailored" validity measures provided by (3.12) and (3.13), and by (3.16) and (3.17), when the information concerning the ability distribution of a target population is more or less available.

### [III.5] Computerized Adaptive Testing

The item information function,  $I_\theta(\theta)$ , has been used in the computerized adaptive testing in selecting an optimal item to tailor a sequential subtest of items for an individual examinee out of the

prearranged itempool. A procedure may be to let the computer choose an item having the highest value of  $I_g(\theta)$  at the current estimated value of  $\theta$  for the individual examinee, which is based upon his responses to the items that have already been presented to him in sequence, out of the set of remaining items in the itempool.

We notice from (3.4) or (3.6) that this procedure is justified from the standpoint of criterion-oriented validity, for the item which provides us with the greatest item information  $I_g(\theta)$  among all the available items in the itempool also gives the greatest values of  $I_g^*(\zeta)$  and its square root, at any fixed value of  $\theta$ .

Amount of test information can be used effectively in the stopping rule of the computerized adaptive testing. A procedure may be to terminate the presentation of a new item out of the itempool to the individual examinee when  $I(\theta)$  has reached an a priori set amount at the current value of his estimated  $\theta$ .

When we have a specific criterion variable  $\gamma$  in mind, it is justified to use an a priori set value of  $I^*(\zeta)$  instead of  $I(\theta)$ . In so doing we can obtain the value of  $I(\theta)$  corresponding to the a priori set value of  $I^*(\zeta)$  for each  $\theta$ , through the formula

$$(3.18) \quad I(\theta) = I^*(\zeta) \left( \frac{\partial \zeta}{\partial \theta} \right)^2,$$

which is obtained from (3.7). Thus it is easy to have the computer to handle this situation, provided that we know the functional formula for  $\zeta(\theta)$ .

## IV Test Validity Measures Obtained from More Accurate Minimum Variance Bounds

When  $\{\partial \zeta / \partial \theta\} = 0$  at some value of  $\theta$ , as is illustrated by a dashed line in Figure 2-2,  $\partial \theta / \partial \zeta$  becomes positive infinity, and so does the item validity measure given by (3.6). This fact provides us with some doubt, for, while we can see that at such a point of  $\zeta$  item validity is high, we must wonder if positive infinity is an adequate measure. It is also obvious from (2.8) that the same will happen to the total test if it includes at least one such item. Our question is: *should we search for more meaningful functions than the item and test information functions?* This topic will be discussed in this section.

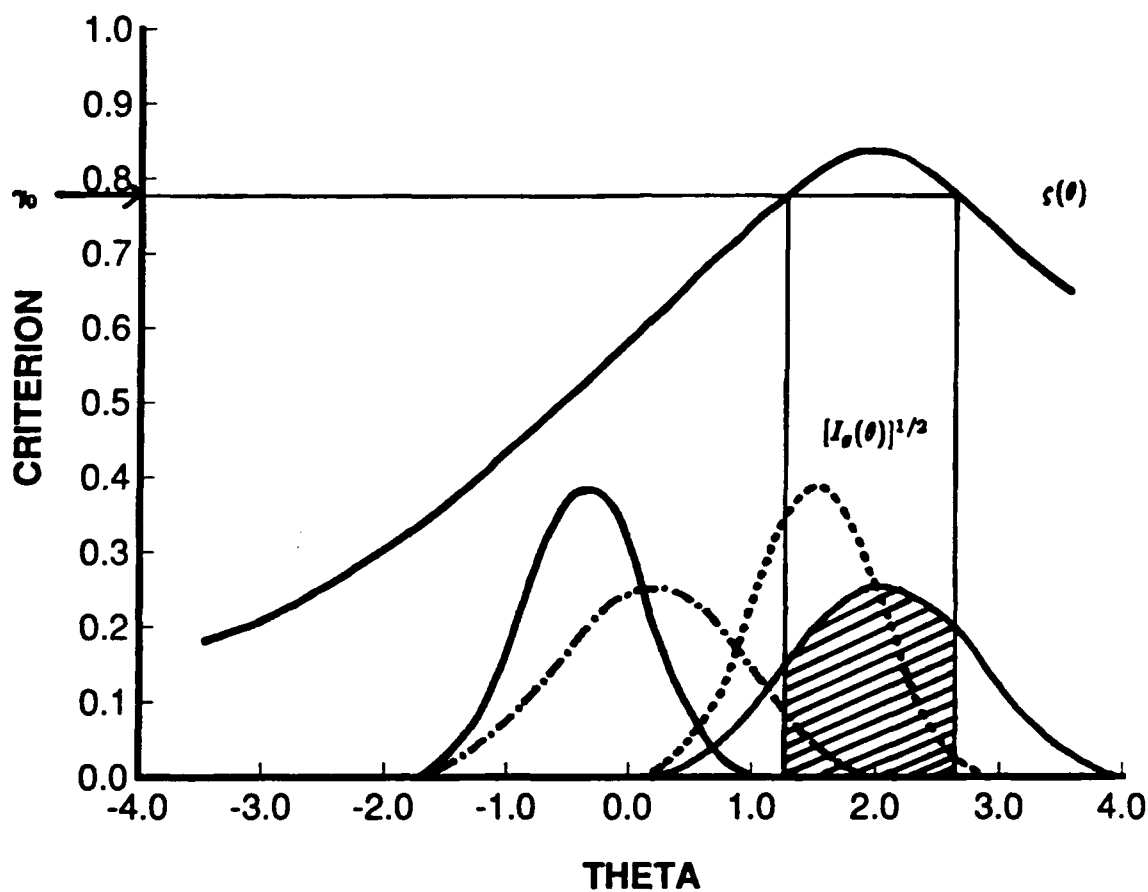
Necessity of the search for a more accurate measure than the test information function becomes more urgent when the performance function,  $\zeta(\theta)$ , is not strictly increasing in  $\theta$ , but is, say, only piecewise monotone in  $\theta$  with finite  $\partial \theta / \partial \zeta$  and differentiable with respect to  $\theta$ , as is illustrated in Figure 4-1. The illustrated performance function is still simple enough, but indicates the trend that after a certain point of ability the performance level in a specified job decreases. This can happen when the job does not provide enough challenge for persons of very high ability levels.

Since  $I^*(\zeta)$  serves as the reciprocal of the conditional variance of the maximum likelihood estimate of  $\zeta$  only asymptotically and there exist more accurate minimum variance bounds for any (asymptotically) unbiased estimator (cf. Kendall and Stuart, 1961), we can search for more accurate test validity measures than the one given by (3.7) by using the reciprocal of the square roots of such minimum variance bounds. Details of this topic will be discussed in a separate paper. Here its brief summary related to validity measures will be given.

Let  $J_{rs}(\theta)$  be defined as

$$(4.1) \quad J_{rs}(\theta) = E \left[ \frac{L_V^{(r)}(\theta)}{L_V(\theta)} \frac{L_V^{(s)}(\theta)}{L_V(\theta)} \mid \theta \right] \quad r, s = 1, 2, \dots, k$$





0.780 0.80 1.00 0.80 0.00  
 DUMMY.DAT, INDDUMM, plotted by LEE HAI GAN

FIGURE 4-1

Example of the Performance Function  $\zeta(\theta)$  Which Is Piecewise Monotone in  $\theta$ .

where

$$(4.2) \quad L_V^{(r)}(\theta) = \frac{\partial^r}{\partial \theta^r} L_V(\theta) = \frac{\partial^r}{\partial \theta^r} P_V(\theta) .$$

Let  $J(\theta)$  denote the  $(k \times k)$  matrix of the element  $J_{rs}(\theta)$ , and  $J_{rs}^{-1}(\theta)$  be the corresponding element of its inverse matrix,  $J^{-1}(\theta)$ . Note that when  $k = 1$  we can rewrite (4.1) into the form

$$(4.3) \quad \begin{aligned} J_{kk}(\theta) &= J_{11}(\theta) = E\left[\left\{\frac{\partial}{\partial \theta} \log L_V(\theta)\right\}^2 \mid \theta\right] \\ &= -E\left[\frac{\partial^2}{\partial \theta^2} \log P_V(\theta) \mid \theta\right] , \end{aligned}$$

and from this, (2.7) and (2.8) we can see that  $J(\theta)$  is a  $(1 \times 1)$  matrix whose element is the test information function,  $I(\theta)$ , itself. A set of improved minimum variance bounds is given by

$$(4.4) \quad \sum_{r=1}^k \sum_{s=1}^k \zeta^{(s)}(\theta) J_{rs}^{-1}(\theta) \zeta^{(r)}(\theta)$$

(cf. Kendall and Stuart, 1961), where  $\zeta^{(s)}(\theta)$  denotes the  $s$ -th partial derivative of  $\zeta(\theta)$  with respect to  $\theta$ . We obtain, therefore, for a set of new test validity measures

$$(4.5) \quad \left[ \sum_{r=1}^k \sum_{s=1}^k \gamma_0^{(s)} J_{rs}^{-1}(\theta(\gamma_0)) \gamma_0^{(r)} \right]^{-1/2} ,$$

where  $\gamma_0^{(s)}$  indicates the  $s$ -th partial derivative of  $\zeta$  with respect to  $\theta$  at  $\zeta = \gamma_0$ .

The use of this new test validity measure will ameliorate the problems caused by  $\{\partial \zeta / \partial \theta\} = 0$ , if we choose an appropriate  $k$ . The resulting algorithm will become much more complicated, however, and we must expect a substantially larger amount of CPU time for computing these measures when  $k$  is greater than unity. Note that (4.5) equals (3.7) when  $k = 1$ .

## V Multidimensional Latent Space

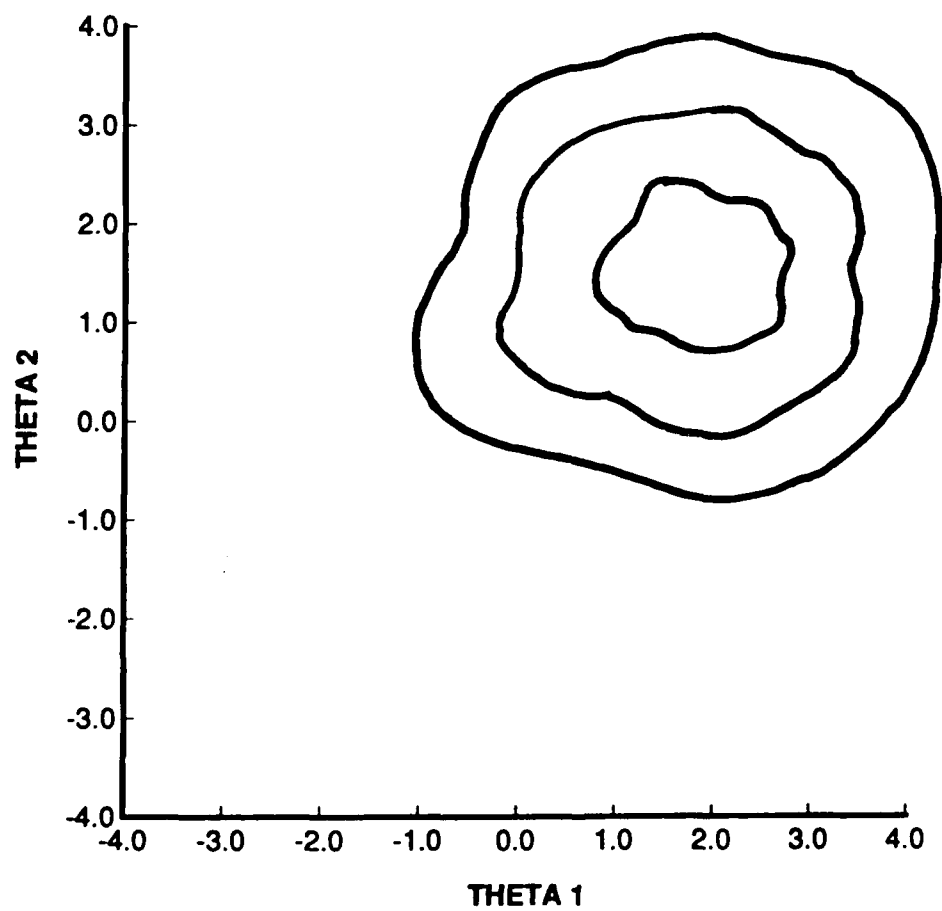
When our latent space is multidimensional, a generalization of the idea given in Section 4 for the unidimensional latent space can be made straightforwardly. We can write

$$(5.1) \quad \theta = \{ \theta_u \}' \quad u = 1, 2, \dots, \eta ,$$

and the performance function  $\zeta(\theta)$  becomes a function of  $\eta$  independent variables. A minimum variance bound is given by

$$(5.2) \quad \sum_{u=1}^{\eta} \sum_{v=1}^{\eta} \frac{\partial \zeta(\theta)}{\partial \theta_u} \frac{\partial \zeta(\theta)}{\partial \theta_v} I_{uv}^{-1}(\theta) ,$$

where  $I_{uv}^{-1}(\theta)$  is the  $(u, v)$ -element of the inverse matrix of the  $(\eta \times \eta)$  symmetric matrix, whose element is given by



1.000 0.80 2.20 0.80 0.90  
 SUMMARY3.DAT, 96048623, plotted by LEE HAI GAN

FIGURE 5-1

Area  $\Omega_0$  for Different  $\gamma_0$ 's in Two-Dimensional Latent Space  
 for a Hypothesised Test.

$$(5.3) \quad I_{uv}(\theta) = E \left[ \frac{1}{L} \frac{\partial L}{\partial \theta_u} \frac{\partial L}{\partial \theta_v} \mid \theta \right]$$

with  $L$  abbreviating  $L_V(\theta)$ , or  $P_V(\theta)$ . The reciprocal of the square root of (5.3) will provide us with the counterpart of (3.7) for the multidimensional latent space. For  $\eta = 2$ , the area  $\Omega_\theta$  may look like one of the contours illustrated in Figure 5-1, depending upon our choice of  $\gamma_0$ , taking the axis for  $\gamma$  vertical to the plane defined by  $\theta_1$  and  $\theta_2$ .

In a more complex situation where both ability and the criterion variables are multidimensional, we must consider the projection of the item information function on the criterion subspace from the ability subspace, in order to have the item validity function for each item, and then the test validity function. It is anticipated that we must deal with a higher mathematical complexity in such a case. The situation will substantially be simplified, however, if the total set of items consists of several subsets of items, each of which measures, exclusively, a single ability dimension and a single criterion dimension.

## VI Discussion and Conclusions

In contrast to the progressive desolution of the reliability coefficient in classical mental test theory and the replacement by the test information function in latent trait models, the issue of test validity has been more or less neglected in modern mental test theory. The present paper proposes some considerations about the validity of a test and of a single item. Effort has been focused upon searching for measures which are population-free, and which will provide us with local and abundant information just as the information functions do in comparison with the test reliability coefficient in classical mental test theory. In so doing, validity indices for different purposes of testing and also those which are tailored for a specific population of examinees are considered.

The above considerations for the item and test validities may be just part of many possible approaches. We may still have a long way to go before we discover the most useful measures of the item and test validities. The aim of the present paper is rather to provide stimulation so that researchers will pursue this topic further, taking different approaches.

## References

- [1] Kendall, M. G. and Stuart, A. *The advanced theory of statistics. Vol. 2.* New York: Hafner, 1961.
- [2] Samejima, F. A use of the information function in tailored testing. *Applied Psychological Measurement*, 1, 1977, 233-247.
- [3] Samejima, F. Constant information model: A new promising item characteristic function. *ONR/RR-79-1*, 1979.
- [4] Samejima, F. Information loss caused by noise in models for dichotomous items. *ONR/RR-82-1*, 1982.
- [5] Samejima, F. Predictions of reliability coefficients of a test using the test information function. *ONR/RR-90-3*, 1990.

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